Bandits and Agents:
How to incentivize exploration?

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EC'15, EC'16
working papers
ongoing work

Motivation: recommender systems

- Watch this movie
- Dine in this restaurant
- Vacation in this resort
- Buy this product
- Drive this route
- See this doctor

- Take this medicine

(medical trials)

- Use these settings

(systems)

Exploration

Recommender system:

- user arrives, needs to choose a product
- receives recommendation (& extra info)
- chooses a product, leaves feedback

consumes info from prior users

produces info for future users

For common good, user population should balance

- ***exploration***: *trying out various alternatives to gather info*
- ***exploitation***: *making best choices given current info*

Example: coordinate via system's recommendations.

Exploration and incentives

Recommender system:

- *agent* arrives, needs to choose a product
- receives recommendation (& extra info)
- chooses a product, leaves feedback

consumes info from prior users

produces info for future users

Agents make decisions based on available info & initial biases

An alternative that seems worse initially may remain unexplored because agents have no incentives to explore it!

How to incentivize agents to explore?

Exploration and incentives

How to incentivize agents to try seemingly sub-optimal actions?

based on agents' biases and/or system's current info)

"External" incentives:

- monetary payments / discounts
- promise of a higher social status
- people's desire to experiment

prone to selection bias;
not always feasible

Exploration and incentives

How to incentivize agents to try seemingly sub-optimal actions?

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"External" incentives:

- monetary payments / discounts
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prone to selection bias;
not always feasible

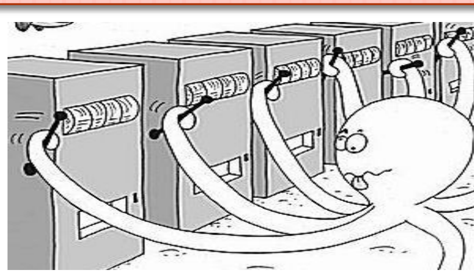
Alternative approach: use ***information asymmetry*** to create ***intrinsic incentives*** to follow system's recommendations

Basic model

- K actions; T rounds
- In each round, a new agent arrives:
  - algorithm recommends an action (& extra info)
  - agent chooses an action, reports her reward $\in [0,1]$
- IID rewards: distribution depends only on the chosen action
- Mean rewards are unknown; common Bayesian prior
- Objective: social welfare (= cumulative reward)

"actions" = "arms"

If agents follow recommendations ⇒ "multi-armed bandits"



classical model in machine learning for explore-exploit tradeoff

Basic model: BIC bandit exploration

How to account for agents' incentives?

Ensure that following recommendations is in their best interest!

Recommendation algorithm is *Bayesian Incentive-Compatible* (BIC) if

$$\mathbb{E}_{\text{prior}}[\text{reward}(a) - \text{reward}(b) \mid \text{rec}_t = a] \geq 0$$

$$\forall \text{round } t, \text{ arms } a, b$$

recommendation in round t

Goal: design ***BIC*** bandit algorithms to maximize **performance**

Can ***BIC*** bandit algorithms perform as well as the best bandit algorithms, BIC or not?

Exploration vs. exploitation

Algorithm wants to balance exploration & exploitation, can choose suboptimal arms for the sake of new info

Each agent is myopic: does not care to explore, only exploits

… based on what she knows:

- common prior
- recommendation algorithm
- algorithm's recommendation (& extra info, if any)

Does not see entire feedback from previous agents

Information asymmetry

- Revealing all info to all agents does not work

> Then algorithm can only exploit ⇒ not good.
> E.g.: can only pick the "prior best" arm.

- So, algorithm needs to reveal less than it knows.
  W.l.o.g., reveal only recommended arm, no extra info

Approach: hide *a little* exploration in *lots of* exploitation.

- Each agent gets "exploitation" with high prob,
  "exploration" with low prob, but does not know which

Related work: multi-armed bandits

- Studied in Econ, OR and CS since 1933
- Most related: IID rewards, with or without a prior
  E.g.: Thompson Sampling, Gittins Index, UCB1 (Auer et al.'02).
- *Best arm prediction*: care about learning rate, not total reward
  E.g.: Even-Dar et al.'02, Goel et al.'09, Bubeck et al.'11.
- Bandits with agents/incentives:

dynamic pricing (E.g.: Kleinberg & Leighton'03, Besbes & Zeevi'09)
ad auctions with unknown CTRs (E.g.: Babaioff et al.'09,'10','13)
dynamic auctions (E.g.: Athey & Segal'13, Bergemann & Valimaki'10)

Related work: BIC exploration in Econ

- Kremer, Mansour, Perry (2014): same model, two arms. Bayesian-optimal algorithm for deterministic rewards, very suboptimal performance for IID rewards
- Frazier, Kempe, J.Kleinberg & R.Kleinberg (2014): payments allowed, agents observe past actions
- Connections to some high-profile work in Economics

  Bayesian Persuasion (Kamenica & Gentzkow: Econometrica'11)

  Strategic Experimentation (Bolton & Harris: Econometrica'99, Keller, Rady & Cripps: Econometrica'05)

Outline

- ✓ Basic model: BIC bandits
- ❑ Our results for BIC bandits
- ❑ Algorithms and key ideas
- ❑ Beyond BIC bandits
- ❑ Discussion and open questions

How to measure performance?

μ_a expected reward of arm a after the prior is realized

For the first t rounds:

- Expected total reward of the algorithm $W(t)$
- Ex-post regret $R_{\text{ex}}(t) = t \cdot (\max \mu_a) - W(t)$
- Bayesian regret $\mathrm{R(t)} = \mathbb{E}_{\text{prior}}[R_{\text{ex}}(t)]$

Can ***BIC*** bandit algorithms attain optimal regret?

Our results: optimal regret

BIC algorithm with optimal ex-post regret for constant #arms:

$$R_{\text{ex}}(T) = O\left(\min\left(\frac{\log T}{\Delta}, \sqrt{T \log T}\right)\right) + c_{\mathcal{P}} \log T$$

For given $(\mu_1, \ldots, \mu_K)$: Δ is the *gap* between best and 2nd-best arm. Optimal for given Δ.

optimal regret in the worst case

Depends on prior $\mathcal{P}$. "Price" for BIC.

Conceptually: *exploration schedule is* ***adaptive*** *to previous observations*

Resolve BIC bandit exploration for constant #arms

Our results: detail-free algorithm

Our algorithm is *detail-free*: requires little info about the prior

- $N > N_0$, where N_0 is a constant that depends on the prior
- $\hat{\mu}$: approx. *min prior mean reward* $\boxed{\mu_{\min} = \min_{\text{arms } i} \mathbb{E}_{\text{prior}}[\mu_i]}$

Extra perks:

- Algorithm does not need to know N_0 and $\mu_{\min}$ exactly
- Agents can have different beliefs, if they believe that:

Our results: black-box reduction

Given arbitrary bandit algorithm $\mathcal{A}$,
produce BIC bandit algorithm $\mathcal{A}'$ with similar performance:

- Bayesian regret increases only by constant factor $c_{\mathcal{P}}$ (which depends only on the prior $\mathcal{P}$).
- Learning rate decreases by factor $c_{\mathcal{P}}$:
  Suppose $\mathcal{A}$ outputs a *prediction* ϕ_t in each round t .
  Then $\mathcal{A}'$ outputs a prediction ϕ'_t distributed as $\phi_{\lfloor t/c_{\mathcal{P}} \rfloor}$.

e.g., predicted best arm

Modular design: use existing $\mathcal{A}$, inject BIC

can incorporate auxiliary info (e.g., prior);
exploration preferences (e.g., arms to favor)

predict beyond the *best arm* (e.g., *worst arm*)

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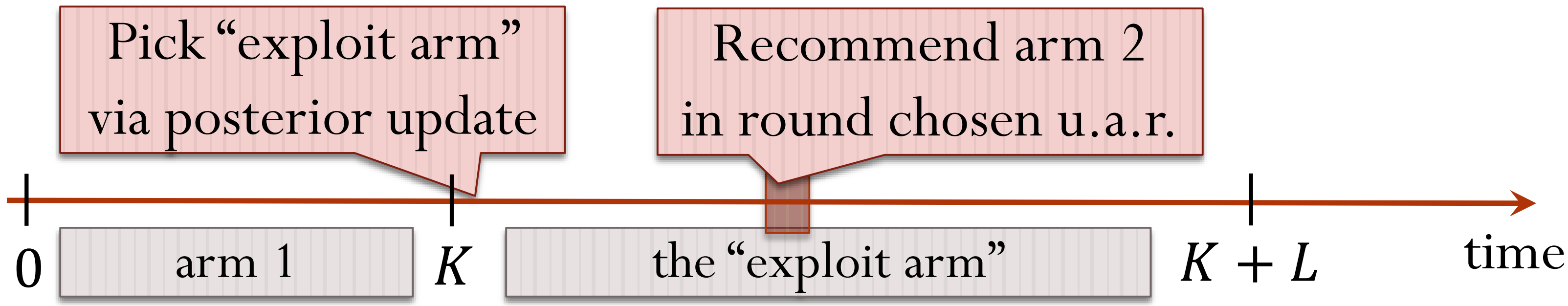
Two arms: $\mathbb{E}_{\text{prior}}[\mu_1 > \mu_2]$

How to sample the other arm?

Hide *exploration* in a large pool of *exploitation*

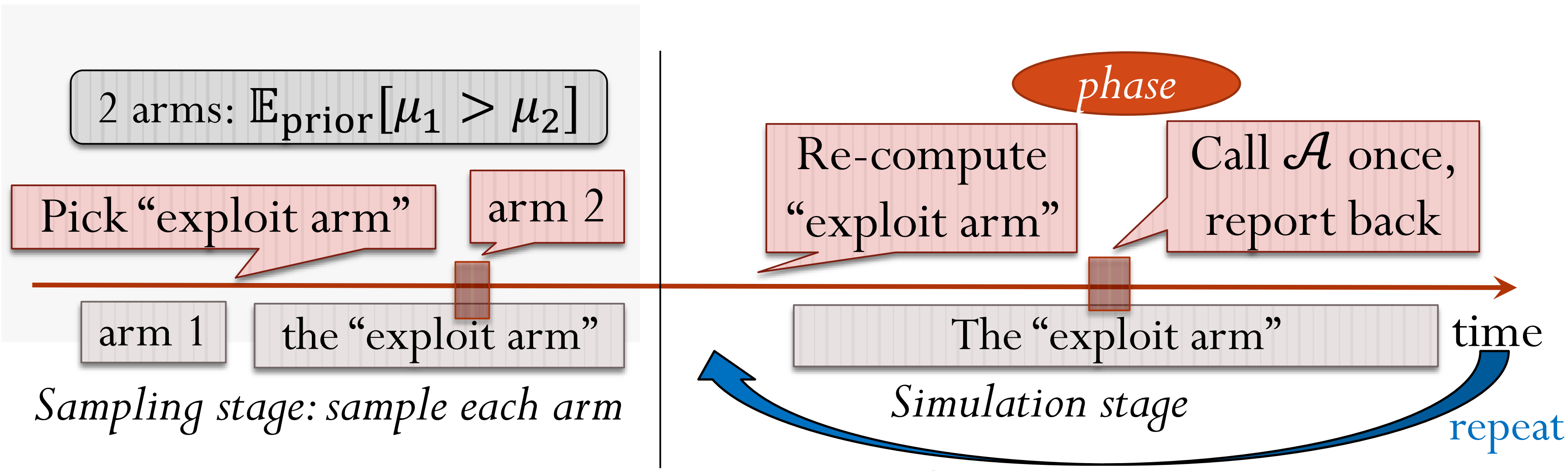


Enough samples of arm 1 ⇒ arm 2 could be the exploit arm!

Agent with rec=arm 2 for exploration does not know it!

Exploration prob. low enough ⇒ follow recommendation.

Black-box reduction from algorithm $\mathcal{A}$



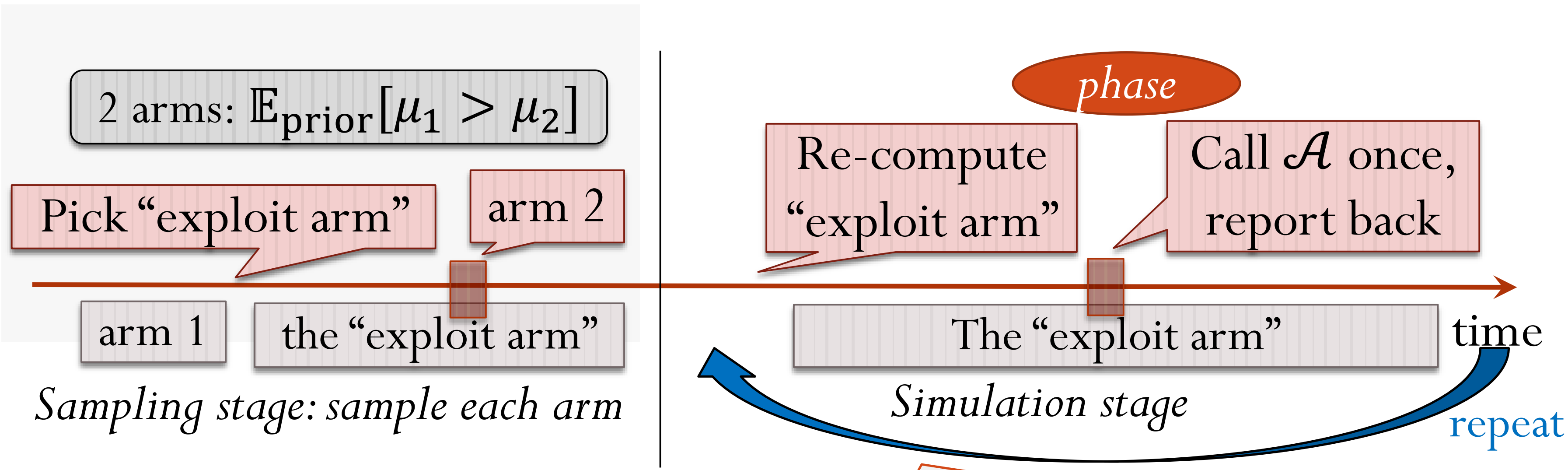
Enough initial samples $\Rightarrow$ any arm could be the exploit arm!

Agent does not know: exploitation or algorithm $\mathcal{A}$?

"Algorithm" prob. low enough $\Rightarrow$ follow recommendation.

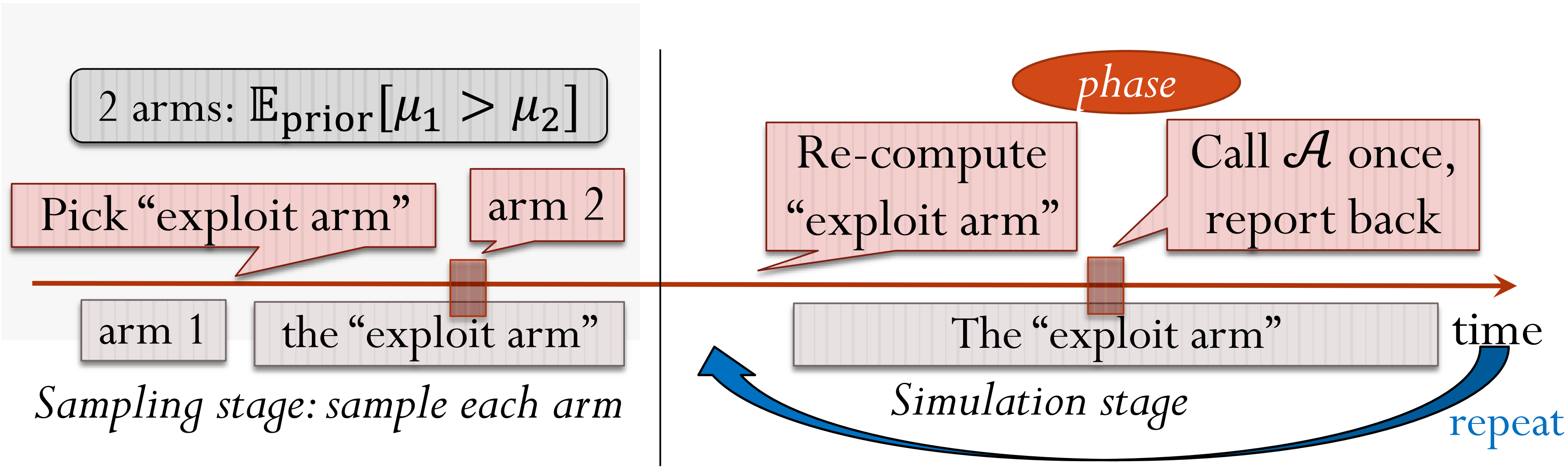
Performance: $\mathbb{E}_{\text{prior}}$[reward] of exploit arm $\geq$ that of $\mathcal{A}$

Black-box reduction from algorithm $\mathcal{A}$



If algorithm $\mathcal{A}$ outputs a ***prediction*** ϕ_t in each round
the reduction outputs the same prediction in all of next phase.
Prediction in round t is distributed as $\phi_{\lfloor t/L \rfloor}$, L = phase length.

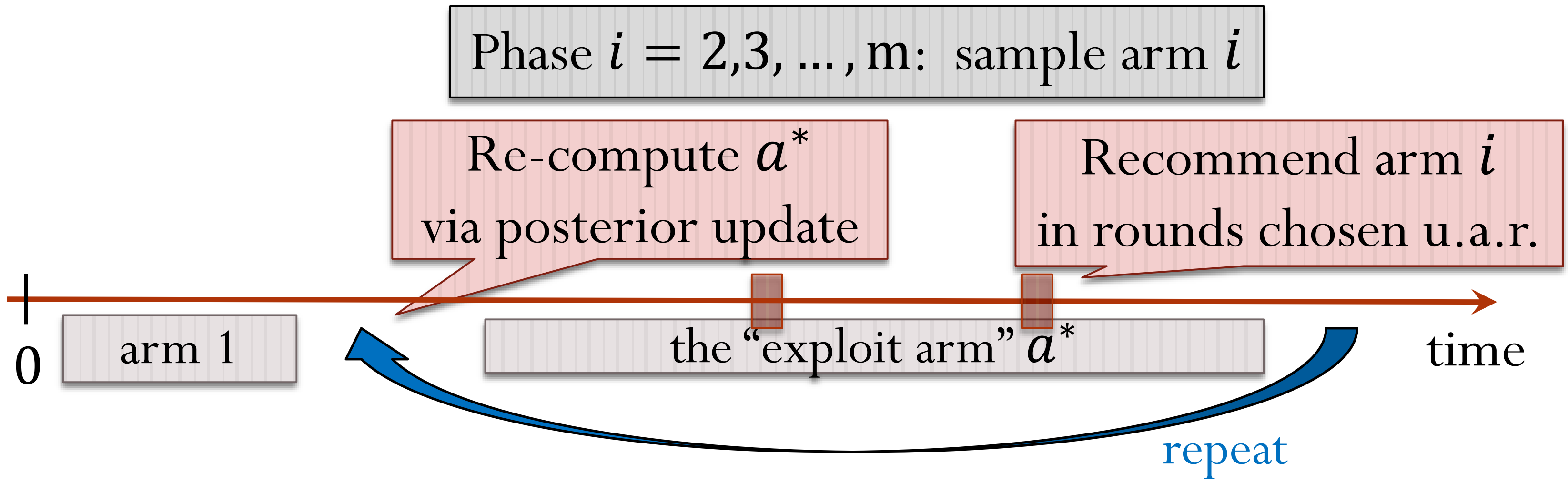
Black-box reduction from algorithm $\mathcal{A}$



Sufficient phase length should not grow over time!
(How low should explore prob. be to convince?)
Analysis of incentives should not depend on algorithm $\mathcal{A}$.

$\mathbb{E}_{\text{prior}}[\mu_1 > \cdots > \mu_m]$

Sampling stage for many arms



Need to make sure that arm i could be the exploit arm!

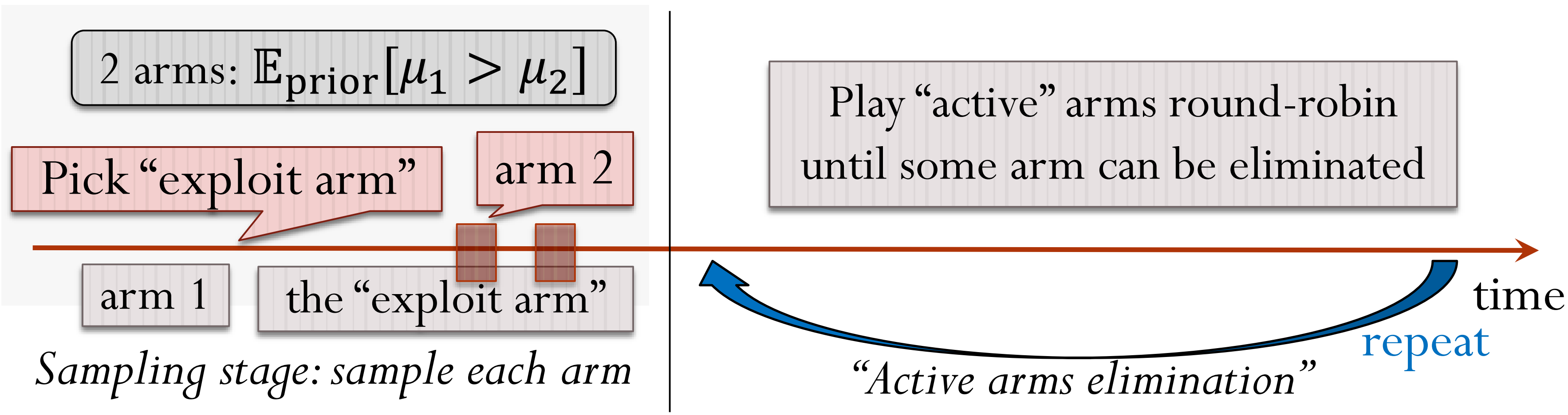
sample each arms $j < i$ enough times

Exploration prob. low enough $\Rightarrow$ follow recommendation.

The detail-free algorithm

Detail-free $\Rightarrow$ cannot use Bayesian update

Ex-post regret $\Rightarrow$ best posterior arm may not suffice



Define "exploit arm" & "elimination condition" via sample averages.

For BIC, connect sample averages to Bayesian posteriors (tricky!).

Enough initial samples $\Rightarrow$ "Active arms elimination" is BIC

Assumptions on the prior

2 arms: $\mathbb{E}_{\text{prior}}[\mu_1 > \mu_2]$

- Hopeless for some priors
  e.g., if μ_1 and $\mu_1 - \mu_2$ are independent.
- Assumption for two arms: for k large enough,

$$\mathbb{P}(\mathbb{E}[\mu_2 - \mu_1 \mid k \text{ samples of arm 1}] > 0) > 0.$$

  Arm 2 can become "exploit arm" after enough samples of arm 1.
  - Necessary for BIC algorithms (to sample arm 2).
    Sufficient for black-box reduction!
- Similar condition for black-box reduction with > 2 arms
  Includes: $\underbrace{\textit{independent priors, bounded rewards, full support on [L,H]}}_{\text{Suffices for the detail-free algorithm}}$

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Black-box reduction with contexts

Our black-box reduction "works" in a very general setting

For each round t, algorithm observes context x_t, then:

- recommends an arm, and (possibly) makes a prediction
- agent chooses an arm, reports her reward & extra feedback

Distribution of reward & feedback depend on arm & context

e.g., customer profile @Amazon

e.g., detailed restaurant reviews

- allows (limited) agent heterogeneity
- incorporates three major lines of work on *bandits*: with contexts, with extra feedback, and with predictions

Setup & result

Policy π: {contexts} → {arms}

Fixed set of policies Π

π^*: best policy in Π

Contextual Bayesian regret

$$R_\Pi(t) = \mathbb{E}_{\text{prior}}[\underbrace{W(t;\pi^*) - W(t;\mathcal{A})}_{\text{total reward}}]$$

Bayesian incentive-compatibility (BIC):

$$\mathbb{E}_{\text{prior}}\left[\mu_{x,a} - \mu_{x,b} \,\middle|\, x_t = x, \text{rec}_t = a\right] \geq 0$$

$\forall$time t, context x, arms a, b

Arms a, contexts x.
Expected reward $\mu_{x,a} \in [0,1]$.

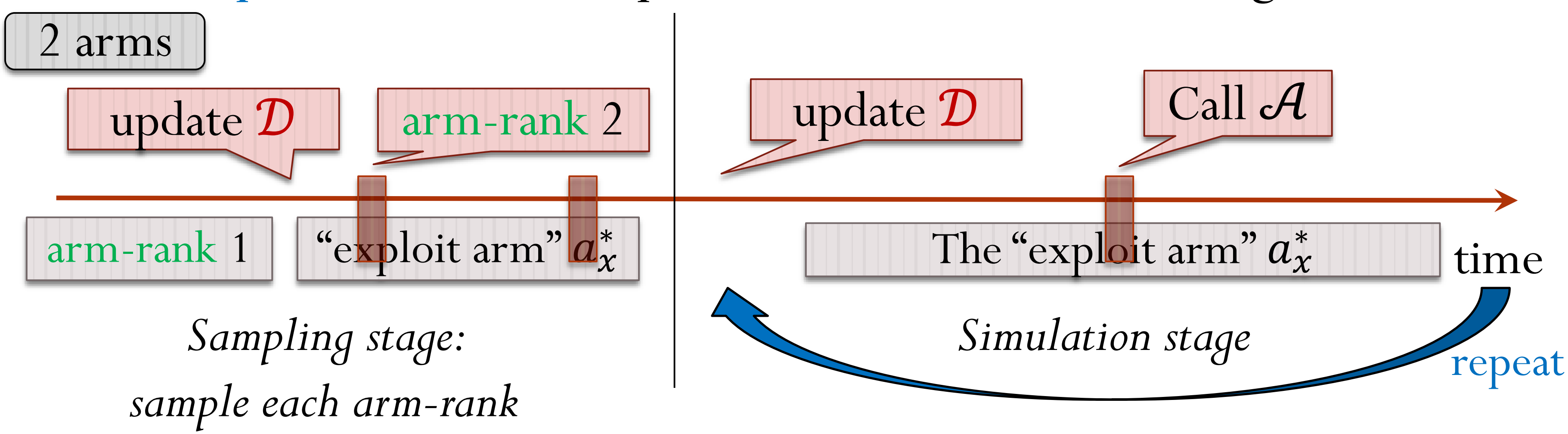
Reduction: bandit algorithm $\mathcal{A} \rightsquigarrow$ BIC bandit algorithm $\mathcal{A}'$ with similar Bayesian regret & prediction quality

Unlike algorithms, our reduction does not depend on:
policy set Π, what is extra feedback, or what is predicted

Algorithm

- Defn: *arm-rank* of arm a given context x is i iff arm a is i-th best given x, according to the prior.
- Key idea: *recommend arm-ranks instead of arms*.
- Maintain a dataset $\mathcal{D}$ of *rank-samples*: $(x, i, \text{reward}, \text{feedback})$.

  Exploit arm a^*_x: best posterior arm for context x given $\mathcal{D}$



BIC bandit games

In each round, a fresh batch of agents plays a game (possibly noisy payoffs, same game in every round)

- algorithm recommends an action to each agent
  E.g., driving directions on Waze
- … chooses a distribution over action profiles
- solution concept: ***Bayesian correlated equilibrium (BCE)***

Which action profiles are "explorable" by a BIC algorithm?

How to explore all of them?

What is the best *BCE* achievable with all explorable info?

How to converge on this *BCE*?

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Connection to medical trials

- Basic design: new drug vs. placebo (blind, randomized)
- “advanced” designs studied & used (adaptive, >2 arms, contexts)
- medical trials is one of original motivations for bandits

- Patients’ incentives: why participate? why take less known drug? Major obstacle, esp. for wide-spread diseases & cheap drugs.
- Medical trial as a BIC recommendation algorithm
  - OK not to give the patients any data from the trial itself
  - extension to contexts and extra feedback very appropriate!

How to *really* convince the patients / model their incentives?

Connection to Systems

- System with many settings/parameters (hidden or exposed) your laptop, smartphone, or facebook feed
- Optimal settings unclear => need for ***exploration***
  - often: settings are hidden, exploration done covertly
- Alternative: expose the settings, let users decide
  - ***explore via incentive-compatible recommendations*** (e.g., the defaults that users can override)
- The version without incentives is understood in theory, but (sort of) open in practice, need to really solve *that* first.

Auxiliary signals

For each agent, algorithm recommends an arm & sends aux. signal

- If algorithm can control whether to send the aux. signal
  - *not sending* is w.l.o.g. if the prior is fully observed & used
  - aux. signal may help for detail-free algorithms
  - cleaner without aux. signals (and we don't use them)
- If algorithm *is required* to send some aux. signals
  - complicated – e.g., revealing full stats does not work!
  - may help to reveal more info than required
  - what *must* and *can* be revealed may depend on application

??

(More) open questions

Optimal dependence on the prior?
Better dependence on #actions?
Action spaces with known structure?
Use exploration that happens anyway?

ML

Fully detail-free algorithms?
Elicit some info from agents?
(ensure they do not lie)

BIC bandit game with
succinct game representation:
better regret, running time?